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BIOLOGICAL CONCEPTS OF THE GROWTH PARAMETERS*

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Abstract

This paper consists of the four short treatises being independent on each other, and the present abstract is divided correspondingly into the following four *main conclusions:*

1. The $\frac{\lg l_{t+1} - \lg l_t}{0.4343} \times l_t$ being called as growth index is a fallacy under cover of mathematics used as a garb. It can not be used as basis when people divide the growth of one generation into several stages and can indicate nothing. Its true features have been *revealed very clearly by a series of self-contradictory calcula*ted results in the article Yellowfin Sole. Hence we should suggest the fish ecology field of all the countries in the world to abrogate the socalled growth index.

2. The relation expression $w_t = a_w l_t^b$, which is used as a sort of mathematical model, depends on the assumption that the fish bodies grow evenly, i. e. body *shape and specific gravity remain unchanged. This assumption is an approximation* for the actual state and is irrelevant to the size of b-values. The fact is not diametrically what the authors of the article Yellowfin Sole said. The former assumption is a deduction reached by reasoning from the "If b = 3".

The growth parameter b in the model $w_t = a_w l_t^b$, which is named as constant of allometry *growth, is the specific value of two growing speeds coming from* the two kinds of biological quantities w_t and l_t of an organism, which increase according to their respective logarithmic values: $b = \frac{d(\log w_t)/dt}{d(\log l_t/dt)}$.

3. The growth parameter t_0, in Von. Bertalanffy's growth functions and the *revision to w_t, is an assumed age. It was born in the assumption that the growth* of average body length (and weight) of one generation during juvenile stage conforms also to the growth law of adult fishes. This assumption is irrelevant to any theories such as biological theory and mathematical theory and so on, but it is put out merely in order to simplify the expression forms of Von. Bertalanffys'

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growth functions and the revision to w_t. Especially, such simplification is indispensable for Von. Bertalanffy's w_t and the revised w_t.

The so-called "theoretical age at which length and weight of fish body equal zero" in the artical Yellowfin Sole, does not exist at all.

4. The growth parameter K, in Von. Bertalanffy's growth functions and the revision to w_t, stands for uniform speed of catabolism in fish body with 1/3 unit weight, or more generally, with 1/b unit weight, and so is called as coefficient of catabolism. This conclusion has been stipulated by the inductive process based on the physiological concept and the mathematical deriving process, in which the former is foundation of the latter, for Von. Bertalanffy's growth functions and the revision to w_t.

The following statement is entirely different to the above contents. In research of differential calculus about curves, K stands for crooked level of a curve at every point within its definition field and so is called as curvature of a curve, which is one of mathematics terms. The curvature K_{l_t}-curve of Von. Bertalanffy's body length growth curve l_t and the curvature K_{w_t}-curve of the revised Von. Bertalanffy's body weight growth curve w_t, have been found and figured in this paper.

The previous two K are respectively in the different research fields. and stand for the different connotations and have the different names. However, the mathematics term "curvature of a curve" was indiscriminately imposed by the authors of the article Yellowfin Sole to the K whose name is originally coefficient of catabolism, so that it is fallaciously called as curvature of the growth curves.

The first order differential equation $\frac{dl_t}{dt} = \frac{Ha_s}{ba_w} - Kl_t$ is a linea relation expression between $\frac{dl_t}{dt}$ and l_t. If the coefficient of l_t which is growing, is called as "growth coefficient", then this coefficient is not K but $(-K)$.

* * * * *

Some problems about biological concepts of the growth parameters will be posed to discuss with the authors of the article entitled as Research In Age And Growth Of Yellowfin Sole (*Limanda aspera*) From Eastern Bering Sea[4] (abbreviated as Yellowfin Sole* under below). Being a reader of the article Yellowfin Sole, I do not ask the authors to agree necessarily with a series of clarifications about these problems in this paper. But I have to clarify clearly the biological concepts of the growth parameters and therefore to present these problems becau-

* The article Yellowfin Sole was published in TRANSACTIONS OF OCEANOLOGY AND LIMNOLOGY. №3, 63—70p. 1989. The page numbers of the quotations noted in this paper were taken from the former 63—70p.

se age and growth of fishes are one of the basic problems in fish population ecology, among which are questions regarding the so-called "growth index" and the growth parameters b, t_0 and K in Von. Bertalanffy's growth functions and the revision to w_t. One of the references quoted in the article Yellowfin Sole, "[2] Chen Dagang, etc., 1984, Preliminary Research In Age And Growth Of *Paralichthys Olivaceus*(T& S) From Yellow Sea And Bohai Sea, And Revising And Discussing Von. Bertalanffy's Growth Functions, JOURNAL OF SHANDONG COLLEGE OF OCEANOLOGY, vol. 14, No.1"[1], was abbreviated as" [2] Chen Dagang, etc., *Paralichthys Olivaceus*" in this paper, and I disdain to ask the authors of the article Yellowfin Sole a question on the so-called "Chen Dagang, etc.".

Now I am obliged to discuss with the authors the following problems:

I. The So-called"Growth Index" Being A Denial To the Relative Growth Rate Is Running In The Opposite Direction To The Ecological Concept Possessed By Growing Fast-Slow Itself

There was a conclusion in part III. Discussion And Conclusion of the article Yellowfin Sole on page 69, excerpted as follows: "2. The stageship of growth of *yellowfin sole: the growth index is used for comparing with growing states in* different terrains and among different fish species, also for dividing growth stages. The calculated results of the growth index in this article indicate that the growth of this fish species can be divided into two stages generally. The period before age 4 is juvenile stage in which growth is obviously faster, while the period after 5 is adult stage in which growth is slower.……" Here is a time vacancy of one year between "before age 4" and "after age 5", so it is necessary to revise the last sentence of the previous quotation according to the measured data about average body length and weight in the table 1 of the article Yellowfin Sole on page 68, into "The period before age 5 is juvenile stage in which growth is obviously faster, while that after age 5 is adult stage in which growth is slower."

The word "growth" in the previous quotation is a joint name of body length growth and body weight growth, therefore, both must be taken simultaneously when people divide the growth of one generation into several stages. For the former conclusion about dividing growth stages, I have to ask the following question: whether was this conclusion based on the calculated results of growth indexes as the quotation said, or based on the calculated results of relative growth rates actually?

It is needed to excerpt the calculated results of relative growth rates C_{lt}, C_{wt} and of growth indexes D_{lt}, D_{wt} from table 1 in the article Yellowfin Sole on page 68. For this, their definitions are firstly listed as follows:

beginning of one year at age t	relative growth rate	growth index
average body length l_t	$C_{lt} \triangleq \frac{l_{t+1}-l_t}{l_t}$	$D_{lt} \triangleq l_t \ln \frac{l_{t+1}}{l_t}$
average body weight w_t	$C_{wt} \triangleq \frac{w_{t+1}-w_t}{w_t}$	$D_{wt} \triangleq w_t \ln \frac{w_{t+1}}{w_t}$

Here I need to explain that the notations of relative growth rates are C_l and C_w in the article Yellowfin Sole. The subscript t is added to the notations C_l and C_w in order to distinguish different age-groups. Besides, the notation D_{lt} of body length growth index is introduced and the definition of body weight growth index D_{wt} is replenished.

Table 1 (a) of female in the article Yellowfin Sole on page 68 is excerpted and replenished as follows:

(1)

age t	♀ average body length l_t(mm)	$C_{lt}\times 100$	very close to D_{lt} < $l_{t+1}-l_t$ (mm/year)		♀ average body weight w_t (g)	$C_{wt}\times 100$	very close to D_{wt} < $w_{t-1}-w_t$ (g/year)	
2	135				50			
		18.52	22.94	25		70	26.53	35
3	160				85			
		20	29.17	32		57.65	38.69	49
4	192				134			
		10.42	19.03	20		38.06	43.22	51
5	212				185			
		6.13	12.62	13		8.11	14.42	15
6	225				200			
		7.56	16.39	17		25.5	45.43	51
7	242				251			
		5.37	12.66	13		19.12	43.92	48
8	255				299			
		3.92	9.81	10		19.06	52.17	57
9	265				356			
		5.28	13.64	14		16.85	55.45	60
10	279				416			
		2.51	6.91	7		12.5	49.00	52
11	286				468			
		4.90	13.67	14		13.68	59.99	64
12	300				532			
		3.33	9.84	10		8.27	42.27	44
13	310				576			
		0.32	0.9984	1		0.69	3.99	4
14	311				580			
		2.89	8.87	9		18.79	99.88	109
15	320				689			

In this table, the values of C_{lt} and C_{wt} when $t=2,3,4$ are obviously bigger than the values of C_{lt} and C_{wt} when $t=5$ to 14. This is a very clear base on which we can divide growth stages of yellowfin sole and draw the following conclusion: the period before age 5 is juvenile stage in which growth is obviously faster, while that after age 5 is adult stage in which growth is slower. I do not know why the authors of the article Yellowfin Sole put aside the very clear basis which had been calculated out by themselves, and did not mention it any longer, but turned to growth indexes, the function of which formed contrast to that of C_{lt} and C_{wt}? There is not any reason for the failure to difine D_{wt} according to the mathematical form to difine D_{lt}. As compared with the function of C_{wt} used to divide growth stages of yellowfin sole, is not the contrast coming from that of D_{wt} excessively great?

The above-mentioned facts indicate the necessity of clarifying what ecological concepts the relative growth rates and the growth indexes express. Here only body length will be taken for discussion, as for body weight, it is similar to body length.

The relative growth rate of body length, $C_{lt}\underline{\Delta}\frac{l_{t+1}-l_t}{l_t}$, is composed by biological quantities l_t and l_{t+1} (average body lengths at the beginning and the end of one year at age t). Such definition form expresses that C_{lt} is specific value of $l_{t+1}-l_t$ (growing quantity of average body length during one year at age t) to l_t (average body length at the beginning of one year at age t), in other words, it denotes how many times the growing quantity $l_{t+1}-l_t$ of average body length within one year at age t is as large as the average body length l_t at the beginning of this year. That is to say that the definition form of C_{lt} (i. e. biological quantity structure) stipulates that C_{lt} denotes annual growing quantity per unit average body length of age t group. Here the ecological concept is very obvious. It is necessary to point out that the "rate" in relative growth rate is neither speed nor coefficient, but specific value or times (is non-dimensional and not a percentage).

The following is to clarify how the body length growth index D_{lt} is composed. From the definition form $C_{lt}\underline{\Delta}\frac{l_{t+1}-l_t}{l_t}$ we can get $l_{t+1}=(1+C_{lt})l_t$. Here $1+C_{lt}=l_{t+1}/l_t$ expresses how many times l_{t+1} is as large as l_t. If $1+C_{lt}$ is equivalently transformed into a power form whose base is e, then the former relation expression between l_t and l_{t+1} can be written as:

$$l_{t+1}=e^{r_t}\cdot l_t, \text{in which } r_t=\ln(1+C_{lt}) \qquad (2)$$

It is necessary to point out that the $1+C_{lt}$ can be equivalently transformed into a power form, base of which is any positive real number noted as a, for instance, $a=10$:

$$l_{t+1}=a^{\log_a(1+C_{lt})}\cdot l_t \qquad (2)_1$$

$$l_{t+1}=10^{\lg(1+C_{lt})}\cdot l_t \qquad (2)_2$$

The definition expression of body length growth index was written in the article Yellowfin Sole as follows:

$$D_{lt}\underline{\Delta}\frac{\lg l_{t+1}-\lg l_t}{0.4343}\times l_t \qquad p67,(17)$$

Contrasting the equalities $(2)_2$ and (2), and noticing $\lg e=0.434294481\cdots\cdots \approx 0.4343$, the following equality can be derived from the definition expression p67, (17) as:

$$D_{lt}\underline{\Delta}\frac{\lg(l_{t+1}/l_t)}{\lg e}\cdot l_t=\frac{\lg(1+C_{lt})}{\lg e}\cdot l_t=[\ln(1+C_{lt})]\cdot l_t=r_t l_t \qquad (3)$$

Equality (3) exhibited such process: the relation expression $l_{t+1}=(1+C_{lt})l_t$ between l_t and l_{t+1} was equivalently transformed into the equality (2) which is a power form of $1+C_{lt}$ whose base is e —first the base of the power form $(2)_2$ was 10, then the base 10 was replaced by e—after that, the product $r_t l_t$ was made by the use of r_t and l_t taken from the equality (2), and used as definition of length growth index D_{lt}. Here an unavoidable question is how to mutiply r_t and l_t? The answer is to use the differential feature of exponential function, there is no other ways except this.

To link the points (t, l_t) on coordinate plane by using following sectionized exponential function curve, as on figure 1.

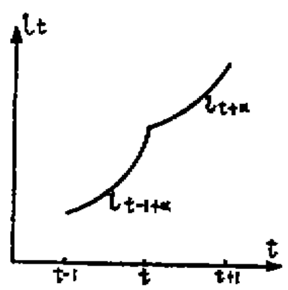

Fig. 1

$$l_{t+\alpha}=e^{r_t\alpha}\cdot l_t, \qquad \begin{matrix}t=1,2,\cdots\cdots\\ 0\leqslant\alpha\leqslant 1\end{matrix} \qquad (4)$$

The derivative function of the equality (4) is

$$\frac{dl_{t+\alpha}}{d\alpha}=r_t\, l_{t+\alpha}, \qquad \begin{matrix}t=1,2,\cdots\cdots\\ 0<\alpha<1\end{matrix} \qquad (4)_1$$

Equality (4) is not differentiable at the moments $t=1,2,\cdots$which are the year-end at age t-1 and also the year-beginning at age t, but its left-derivative and right-derivative are respectively as follows:

$$\left(\frac{dl_t}{d\alpha}\right)_- \triangleq \lim_{\alpha\uparrow 1}\frac{dl_{t-1+\alpha}}{d\alpha} = \lim_{\alpha\uparrow 1}(r_{t-1}l_{t-1+\alpha}) = r_{t-1}l_t$$

$$\left(\frac{dl_t}{d\alpha}\right)_+ \triangleq \lim_{\alpha\downarrow 0}\frac{dl_{t+\alpha}}{d\alpha} = \lim_{\alpha\downarrow 0}(r_t l_{t+\alpha}) = r_t l_t, \qquad t=1,2,\cdots\cdots(4)_2$$

At the right end of the second equality of $(4)_2$ the product r_tl_t appeared at last, which was called as body length growth index by the authors of the article Yellowfin Sole. But very sorry to say, the formal logic exhibited in the equalities from (4) to $(4)_2$ has not any connection with actual growing process of body length. Can the equality (4) be used as an approximate description of growing process of average body length during one year at age t? Such approximate description is useless. Can the second equality of $(4)_2$, i. e. the right-derivative be adopted at $t=1,2,\cdots\cdots$? Then, what is the reason for not adopting the first equality of $(4)_2$, i. e. the left-derivative? What can $D_{lt}\triangleq r_tl_t$ called as body length growth index actually indicate?

The ecological concept possessed by fast-slow of body length growth include two aspects, one is the magnitude of annual growing quantity of average body length and another is that of unit average body length, here one year is taken as a time unit. The latter, i. e. relative growth rate of body length, is used to compare with fast-slow of body length growth among different age-groups and is also the base for dividing growth stages. Especially, the comparison of growing fast-slow of body lengths among different age-groups which have the same annual growing quantities of average body length can be only based upon the relative growth rate of body length. It is to say that an age-group is growing faster if its annual growing quantity equaling others comes from a smaller average body length at the year-beginning. In contrast to this, the length growth index D_{lt} used to describe growing fast-slow is running in the opposite direction to the ecological concept possessed by above-mentioned growing fast-slow itself. I might as well excerpts ome parts as follows from the previous table (1) and suggest the authors of the article Yellowfin Sole to look into them with carefulness.

t	l_t	$l_{t+1}-l_t$	C_{lt}	D_{lt}
5	212	13	6.132	12.617
6	225			
7	242	13	5.372	12.663
8	255			

t	l_t	$l_{t+1}-l_t$	C_{lt}	D_{lt}
9	265	14	5.283	13.643
10	279			
11	286	14	4.895	13.668
12	300			

t	l_t	$l_{t+1}-l_t$	C_{lt}	D_{lt}
8	255	10	3.922	9.809
9	265			
12	300	10	3.333	9.837
13	310			

$(1)_1$

t	w_t	$w_{t+1}-w_t$	C_{wt}	D_{wt}
4	134			
		51	38.060	43.217
5	185			
6	200			
		51	25.5	45.427
7	251			

$(1)_2$

What can the readers read in the face of a series of self-contradictory calculated results that were put up there by the authors? Growth means body length growth and body weight growth, why not calculate the values of the body weight growth index D_{wt}? The cause is not only that D_{wt} can not be used as base for dividing growth stages even more than D_{lt}, but is also that the table $(1)_2$ reveals more self-contradiction than the table $(1)_1$.

It is necessary to research the following function because of the questions exposed in the tables $(1)_1$ and $(1)_2$. Assuming that annual growing quantity $l_{t+1}-l_t$ of l_t being average body length at the year-beginning is a constant noted as $a>0$, thus the following expression with the parameter a of D_{lt} would be derived from the equality (3):

$$D_{lt}=\left[\ln\left(1+\frac{a}{l_t}\right)\right]\cdot l_t,\quad a>0 \qquad (3)'$$

It is easy to prove that D_{lt} is a monotonous increasing function with an independent variable l_t and tends rapidly to a with l_t increasing, explained by figure 2(1) and the following appended table:

l_t	$a/6$	$a/3$	$a/2$	a
D_{lt}	$0.3243a$	$0.4621a$	$0.5493a$	$0.6931a$
l_t	$5a$	$10a$	$20a$	$310a$
D_{lt}	$0.9116a$	$0.9531a$	$0.9758a$	$0.9984a$

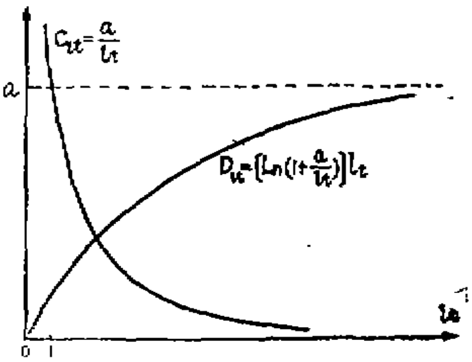

Fig. 2(1)

In the table (1), the D_{lt}-value of every age-group is very close to the corresponding annual growing quantity of average body length. It results only from the fact that the average body lengths l_t at these year-beginning are 5 to 310 times as large as the corresponding annual growing quantities, but not from that D_{lt} can indicate nothing. The relative growth rate of body length, $C_{lt} \triangleq \frac{l_{t+1} - l_t}{l_t}$, used as a supplement for annual growing quantity $l_{t+1} - l_t$ of average body length, displays especially its necessity when different age-groups have the same $l_{t+1} + l_t$. That is to say that an age-group is growing slower if its annual growing quantity equaling others comes from a bigger l_t. This is ecological concept possessed by growing fast-slow itself. However, this bigger l_t while corresponding to a smaller C_{lt}, on the contrary, corresponds to a bigger D_{lt}. It is inevitable to draw the conclusion that this age-group is growing faster if based on D_{lt}. This fact has clearly indicated that D_{lt} is a denial to C_{lt}. With l_t increasing, C_{lt} decreases monotonously and tends to zero; while D_{lt} increases monotonously and tends to $a = l_{t-1} - l_t$, so went around in a large circle, it goes back to annual growing quantity of average body length at last. The differences in size among the calculated values of D_{lt} listed in the article Yellowfin Sole are only some confusing results, here the authors mix up l_t and $l_{t+1} - l_t$ for every age-group with the mathematical form going against the ecological concept possessed by growing fast-slow itself. The true features of D_{lt} at last are revealed among the different age-groups which have the same $l_{t+1} - l_t$. Is it possible that the authors of the article Yellowfin Sole have not found this? After all, these results were calculated by yourselves although some of them have been wrong (refer to the table (1)′).

t	♀ l_t	$l_{t+1} - l_t$	C_{lt}	D_{lt}	♂ w_t	C_{wt}
2					45	wrong, should be 99.56 95.56
3					88	
4	192	wrong, should be 26 20				
5	212		wrong, should be 6.77 6.13			
6	225					
9					267	wrong, should be 22.96 32.96
10					355	
13	310			wrong, should be 2 1		
14	311					

(1)′

Can the growth index be retrieved in some other ways? For example, we adopt its reciprocal so that the $1/D_{lt}$ decreases monotonously like C_{lt} with l_t increasing for the same $a = l_{t+1} - l_t$. But it is really a pity that $\lim_{l_t \to \infty} \frac{1}{D_{lt}} = \frac{1}{a}$, and in such limiting processes for every value of l_t a bigger value of a corresponds to a smaller value of $1/D_{lt}$, as shown in figure 2(2).

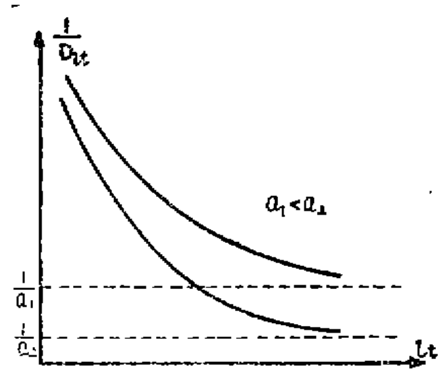

Fig. 2(2)

Ⅱ. $W_t = a_W l_t^b$, "If b = 3, Then It Denotes That The Fish Bodies Grow Evenly, i. e. Body Shape And Specific Gravity Remain Unchanged." In Former Proposition, The "If b = 3" And The "Then……" Between Which There Is Not Originally Any Logical Connection, But They Were Dragged Stiffly To The Same Place To Fabricate A Causation.

In the article Yellowfin Sole on page 64 the ecological concept of b was stated as follows: "The b-value in the relation expression $w_t = a_w l_t^b$ is relation index between body length and weight. If b = 3, then it denotes that the fish bodies grow evenly, i. e. body shape and specific gravity remain unchanged. The values of index b of hardbone fishes are generally from 2.5 to 4.0. The b-value of yellowfin sole is just in this range and is close to 3. It denotes that the growth of yellowfin sole is in accordance with the assumption about even growth." Here I have to ask the authors the following two questions:

1). "The fish bodies grow evenly, i. e. body shape and specific gravity remain unchanged". Whether is this statement actually based on b = 3 as its sign, or is an assumption irrelevant to the size of b-value, on which the power function model $w_t = a_w l_t^b$ was induced and summerized based?

2). What is ecological concept of the growth parameter b?

Need to consult the exposition in the reference" [2] Chen Dagang, etc., *Paralichthys Olivaceus*" on pages 105-107. This is about the relation expression $w_t = a_w l_t^b$ between average boby length l_t and weight w_t at every moment t during adult stage of all individuals of one generation of a fish population.

The relation expression "assumed that the individuals of single fish species about every kind of body shape (spindle, flat, stick, etc.) are the similar geometric solids and their growth is a similar enlargement. 'In the sense taking average for all individuals at the same moment t', the volume of the geometric solid is b-th power of the body length ($b>2$, is an undetermined constant), and the body weight equals the volume timed by a revising coefficient a_w which relates to width, height, density of fish body (and selected units of body length and weight)."

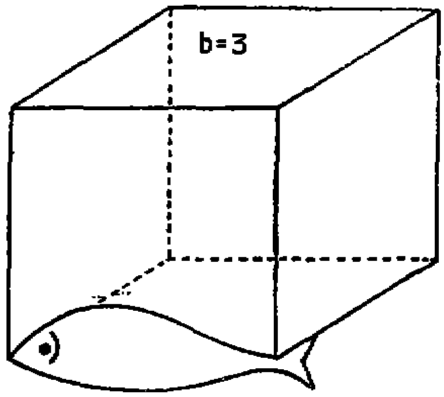

Fig. 3

"In this treatise, the exponent 3 in the assumed condition that was depended on deriving Von. Bertalanffy's growth functions, was revised as an undetermined constant b. Such revision is based on the power function model $w_t = a_w l_t^b$ which is generally adopted to describe the relation between l_t and w_t, but the relation between the function expressions* (2) and (3)′ is in contradiction to this model although there is only a little disparity between 3 and b-value of any one among many fish species [18]. For Beverton-Holt yield model, it is not necessary to use uniformly 3 as approximate value of b……"

"b, which is named as constant of allometry growth, is the specific value of two growing speeds coming from the two kinds of biological quantities w_t and l_t of an organism, which increase according to their respective logarithmic values: $b = \frac{d(\log w_t)/dt}{d(\log l_t)/dt}$. The base of 'log' in former expression can be any positive real number which is generally e or 10 for convenience in research or calculation."

Like any biomathematical models, $w_t = a_w l_t^b$ is an approximate description for the mutual relation between objectively existent biological quantities and their changing laws. This model did not result from mathematical derivation, but is a result that people get by inducing and summarizing lots of observed-measured data

* $l_t = L_\infty[1 - e^{-K(t-t_0)}]$ (2)
$w_t = W_\infty[1 - e^{-K(t-t_0)}]^3$ (3)′ i. e. Von. Bertalanffy's growth functions.

of l_t and w_t for many sorts of fish species, and therefore it is a data-empirical model. This model may be used to describe approximately the relation between l_t and w_t at every moment t during adult stage of one generation. In the course of l_t and w_t increasing with age t, the relation between them remains unchanged, that is, the power exponent b remains unchanged——the volume of the geometric solid is b-th power of the body length and growth is a similar enlargement; the revising coefficient a_w remains unchanged——the body weight equals the volume timed by a revising coefficient a_w which relates to the width, height and density of fish body. Such two "remains unchanged" are just above-mentioned "The fish bodies grow evenly, i. e. body shape and specific gravity remain unchanged." But in fact, there are some differences in environment conditions among various years throughout the generation's life, and connecting with alternation of four seasons all every year round, the fishes would still undergo different living stages such as generating, overwintering, migrating, etc. These causes will inevitably affect body shape and specific gravity (density). The remark "The fish bodies grow evenly, i. e. body shape and specific gravity remain unchanged" is just a kind of approximation to actual body shape and specific gravity that are changing with continuous age t of fish increasing. Such approximation is indispensable in order to induce and summarize the biomathematical model $w_t = a_w l_t^b$. It is all the same whether b equals 3 or not. Moreover in practical work, it is difficult to collect data of body lengths and weights of one generation at every year-beginning, so people use generally data of body lengths and weights of various age-groups taken from catches. The fishes of these age-groups were caught respectively from various generations which have different living experiences. For the actually measured values of l_t and w_t of a fish species, both the constants b and a_w are a sort of many years' averaging results engendered by linearizing step $\ln w_t = \ln a_w + b \ln l_t$ and using Least-square method. This is the actual meaning of the assumption "The fish bodies grow evenly, i. e. body shape and specific gravity remain unchanged."

The linear-relation expression $\log w_t = \log a_w + b \log l_t$ indicates that any growing quantity of w_t in its own logarithmic value is b times as big as the corresponding growing quantity of l_t in its own logarithmic value, hence b is called as constant of allometry growth. It is also allometry growth when $b=3$, while called as synchronous growth only when $b=1$. For the sorts of hardbone fishes, there are not the synchronous growth and their biomathematical models $w_t = a_w l_t^b$ in which the b-values ranging from 2.5 to 4.0 all depend on the assumption "The fish bodies grow evenly, i. e. body shape and specific gravity remain unchanged" without exception. It is not that only yellowfin sole whose b-value is close to 3 relies on this assumption. Moreover, this assumption does not depend on $b=3$, There is not any logical connection between this assumption and the values of b, and the causati on which is the so-called "If $b=3$, then……" is a sheer fabrication.

Ⅱ. "t_0——Theoretical Age At Which Length And Weight Of Fish Body Equal Zero," There Is Not Any Theory To Be The Base Of This Fabricated Proposition

What is the t_0 in Von. Bertalanffy's growth functions and the revision to w_t? "t_0——theoretical age at which length and weight of fish body equal zero". The article Yellowfin Sole on page 65 declared by this proposition to the readers that t_0 is an age which has been researched based on some theory, and at this age length and weight of fish body equal zero. Do such theory and such theoretical age exist really? It is general biological knowledge that development of a fish begins from an embryo of a fertile egg. Then, what kind of biological theory can be used to research embryoes of fertile eggs in drawing the conclusion that embryoes have no size, i. e. no length and no weight when they begin to develop?

Von. Bertalanffy's body length growth function

$$l_t = L_\infty[1 - e^{-k(t-t_0)}], \qquad t_1 \leqslant t \leqslant t_\lambda \tag{4}$$

is the solution of the following first order differential equation with an initial value condition:

$$\begin{cases} \dfrac{dl_t}{dt} = K(L_\infty - l_t) & (5)_1 \\ l_t|_{t=t_1} = l_1, \quad t_1 \leqslant t \leqslant t_\lambda & (5)_2 \end{cases} \qquad (5)$$

The general solution of the equation $(5)_1$ is the following function family with a positive parameter c and the corresponding curve family that are shown in equality $(4)_1$ and in figure 4.

$$l_t = L_\infty - ce^{-Kt}, \quad c>0 \tag{4$_1$}$$

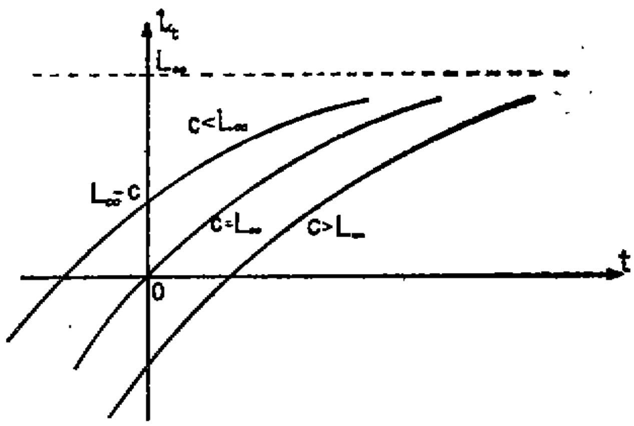

Fig. 4

The t_1 in the function expression (4) and in $(5)_2$ is an age at which fishes of one generation have developed into overall sex-mature basically for the first time. It signs that this generation has developed into adult stage in which physiological state of fishes is basically stable. Such stability is physiological base on which people can make the previous assumption "The fish bodies grow evenly, i. e. body shape and specific gravity remain unchanged." Although, the environment condition would vary continuously during the period after t_1 until t_λ, here the t_λ is an age at which fishes die as a result of senile decay, and moreover connectiong with alternation of four seasons all every year round, the fishes would still undergo different living stages such as generating, overwintering, migrating, etc.

Putting the initial value in $(5)_2$ into the general solution $(4)_1$, the corresponding $c=(L_\infty-l_1)e^{kt_1}$ is found. From this, one special solution of the differential equation (5) is determined. This special solution is a section within l_t's definition field of the curve which is one among the general solution curve family $(4)_1$. This curve section is shown in equality $(4)_2$ and passes initial value point (t_1, l_1).

$$l_t=L_\infty-(L_\infty-l_1)e^{-K(t-t_1)},\quad t_1\leqslant t\leqslant t_\lambda \qquad (4)_2$$

The definition field $t_1\leqslant t\leqslant t_\lambda$ in $(5)_2$ came from physiological concept. Within this range of age the assumption "The fish bodies grow evenly, i. e. body shape and specific gravity remain unchanged" is approximately suitable. So it can be suggested by people under the many years' averaging meaning and is totally irrelevant to the "If $b=3$".

Studying on juvenile fishes and that on adult fishes belong separately to two different research domains that are "juvenile ecology" and "reproductive stock ecology". Von. Bertalanffy's body length growth function, i. e. the special solution $(4)_2$ of the differential equation (5), and the body weight growth function to be derived from this with the aid of the relation expression $w_t=a_w l_t^b$ have no ecological meaning on the left of t_1 on t-axis, i. e. juvenile stage. There is no t_0 originally in the expressions of the two functions, and also the so-called "theoretical age at which length and weight of fish body equal zero" and some ages according to such logic, at which length and weight of fish body equal negative numbers do not exist at all.

"[2] Chen Dagang, etc., *Paralichthys Olivaceus*" on page 106 says, "If the law expressed by the equality $(2)_1$ (i. e. the function expression $(4)_2$ in this paper, the same as below——noted by the writer of this paper) was prolonged from adult stage outwards, i. e. if we assume that the growth of average body length of one generation during juvenile stage conforms also with such law, then we would extrapolate out an assumed age t_0 which corresponds to zero length, i. e. $l_{t_0}=0$. The t_0 is an intersection point at which the curve described by the equality $(2)_1$ was prolonged left up to crossing t-axis:

$$t_0 = t_1 + \frac{1}{K}\ln\left(1 - \frac{l_1}{L_\infty}\right) \qquad \text{p106, (4)}$$

Obviously, $t_0 < t_1$. Substituting the equality (4) on page 106 for t_1 in the equality $(2)_1$ (i. e. the special solution $(4)_2$ in this paper, see the note above), in other words, using $l_{t_0}=0$ as assumed initial value condition in which the t_0 is determined by the above equality (4), then the following simplified form of the equality $(2)_1$ can be obtained: $l_t = L_\infty[1-e^{-K(t-t_0)}]$, $t_1 \leqslant t \leqslant t_L$," which is the equality (4) in this paper. It is a simplification of the function expression $(4)_2$ in mathematical form, and is still the original section within the definition field $t_1 \leqslant t \leqslant t_L$ of the original curve which is determined by the actual (measured) initial value condition $l_{t_1}=l_1$. The origin of t_0 stipulates that t_0 has not any theoretical concept and is only a result of assuming that the growth of average body length of one generation during juvenile stage conforms also with such law (of adult fishes), so t_0 should be called as assumed age. This assumption is put out only in order to simplify expression forms of Von. Bertalanffy's growth functions which were used as a sort of mathematical model, but it is irrelevant to mathematical theory. The authors of the article Yellowfin Sole while discussing so-called "theoretical age at which length and weight of fish body equal zero", can not have any theoretical basis, biological basis or mathematical basis. In fact, every curve among the general solution curve family $(4)_1$ of the differential equation $(5)_1$ intersects with t-axis and has also a part below t-axis where l_t's values are negative. This fact is shown very clearly in figure 4.

It is necessary to emphasize that to determine t_0 according to the equality (4) on page 106 in the reference "[2] Chen Dagang, etc., *Paralichthys Olivaceus*" is prerequisite of $l_{t_0}=0$ being used as the assumed initial value condition. The $t_0=-1.587$ in the article Yellowfin Sole was calculated by using the initial value condition $l_t|_{t=2}=135$mm according just to this equality (4). It is a testimony for this that body length growth curve $l_t=359[1-e^{-0.1315(t+1.587)}]$ (of female) passes initial value point (2,135), here $t_1=2$. (Please take notice to that the female fishes of yellowfin sole do not develop into sex-mature at age 2 but the sex-mature begins until age 5[2][3]). For the $l_2=135$mm and $w_2=50$g of yellowfin sole's female individuals at the year-beginning at age 2, the growing time of these individuals from birth to the year-beginning at age 2 is actually two years, but one would be $2-(-1.587)=3.587$ years if growth begins from the assumed age $t_0=-1.587$. Thus, if growth of average body length (and weight) before age 2 is assumed to accord with the equality $(4)_2$ or equivalent equality (4) in this paper, i. e. the growing law after age 2 expressed by Von. Bertalanffy's growth functions, then it is slower than actual growth. As a result, it can not be used that are the parts within the interval $[t_0, t_1)$ into which these two growth curves prolonged to the left. The t_c in Beverton-Holt yield model, being an age at first capture with given fishing-gear, is generally stipulated to be not less than t_1.

The $t_0 = -1.587$ in the article Yellowfin Sole has been calculated from $-1.5869 = 2 + \frac{1}{0.1315}\ln\left(1 - \frac{135}{359}\right)$. This fact testifies obviously that this t_0-value was found by the authors according to the equality (4) in the reference "[2] Chen Dagang, etc., *Paralichthys Olivaceus*" on page 106 and using the initial value condition $l_t|_{t=2} = 135$mm. But on the contrary, the article Yellowfin Sole on page 65 recommended to the readers the following method that its authors themselves do not know how to use:"the t_0 is evaluated according to the equation of regression straight line: $\ln(L_\infty - l_t) = (\ln L_\infty + Kt_0) - Kt_0 \cdots\cdots (d)$". Obviously, there is actually no t_0 to be evaluated in the equation (d) because $Kt_0 - Kt_0 \equiv 0$. This is possibly the authors' a slip of the pen which appears at the situation of crucial importance. This equation should be written as: $\ln(L_\infty - l_t) = (\ln L_\infty + Kt_0) - Kt \cdots\cdots (d)$. It is an equivalent transform of Von. Bertalanffy's body length growth function, and it also can be transformed into the following form to be still noted as equation (d) and the following equality $(6)_1$. The mathematical form of $(6)_1$ is the same as the equality (4) in the reference "[2] Chen Dagang, etc., *Paralichthys Olivaceus*" on page 106:

$$\ln\left(1 - \frac{l_t}{L_\infty}\right) = Kt_0 - Kt \qquad \text{p65, (d)}$$

$$t_0 = t + \frac{1}{K}\ln\left(1 - \frac{l_t}{L_\infty}\right) \qquad (6)_1$$

The above-mentioned equality (4) on page 106 uses only (t_1, l_1) marking the beginning of adult stage as the initial value point, but the equality (d) on page 65 and its equivalent transform $(6)_1$ use (t, l_t), $t = t_1, t_1 + 1, \cdots, t_\lambda - 1$ one after another as the initial value point. In practical work, one certainly may use several measured (t, l_t) of younger adult fishes as initial value points, for example $t = t_1, t_1 + 1, t_1 + 2$, etc., which are respectively substituted into equality $(6)_1$ to evaluate the corresponding values of t_0, then the arithmetic mean of these t_0-values is adopted.

Note the maximum age in the specimens as t_m. For age t from t_1 to t_m, if the measured (t, l_t) are substituted one after another into the equality $(6)_1$ to evaluate the corresponding t_0-values, then final t_0-value is adopted as:

$$t_0 = \frac{1}{t_m - (t_1 - 1)} \sum_{t=t_1}^{t_m} \left[t + \frac{1}{K}\ln\left(1 - \frac{l_t}{L_\infty}\right)\right] \qquad (6)_2$$

Then, we shall show how to realize "the t_0 is evaluated according to the equation of regression straight line……(d)" which was recommended to the readers by the authors of the article Yellowfin Sole.

The general model of linear regression with one independent variable in common mathematical statistics textbooks is $y=a+bx$. Here let $x=t$, $y=\ln(1-l_t/L_\infty)$, slope $b=-k$, intercept $a=kt_0$ according to the requirements of the model "p65, (d)". Then use measured (x_i, y_i), $i=1-n$, that is, $(t, \ln(1-l_t/L_\infty))$, $t=t_1-t_m$ to evaluate the Least-square estimated values of a and b. Finally obtain $K=-b$, $t_0=a/(-b)$.

Previous process estimating the values of K and t_0 according to the model "p65, (d)" is beginning when L_∞ has been evaluated, here L_∞ is called as limited average body length. I must emphasize that L_∞ in the model "p65, (d)" is known, otherwise this model is useless. For this, the article Yellowfin Sole on page 65 recommends to the readers the following calculating steps: "First, using measured values of average boby lengths of various age-groups, L_∞ and K are evaluated according to the Ford-Walford plot of l_{t+1} against l_t which is expressed by the relation expression of regression straight line: $l_{t+1}=L_\infty(1-e^{-k})+e^{-k}l_t$……(c). Here L_∞ is ultimate asymptotic body length and K is curvature of the growth curves. Then W_∞, which is ultimate asymptotic body weight, is evaluated according to the relation between body length and weight. Finally, the t_0 is evaluated according to the equation of regression straight line: $\ln(L_\infty-l_t)=(\ln L_\infty+Kt_0)-Kt$……(d)". It was written very clearly that before "Finally, the t_0 is evaluated according to…… (d)" there is already "First,……L_∞ and K are evaluated according to ……(c)." Here I excerpt the calculated results according to the model (c) from the article Yellowfin Sole:

$$\text{First step } (♀): \quad l_{t+1}=L_\infty(1-e^{-k})+e^{-k}l_t,\ t=t_1—t_n \qquad \text{p65,(c)}$$
$$K=0.1315, L_\infty=359\text{mm} \qquad (c)'$$

and write the calculated results according to the model (d), which was done by the writer of this paper:

$$\text{Second step } (♀): \ln\left(1-\frac{l_t}{359}\right)=Kt_0-Kt, t=t_1——t_n \qquad \text{p65,(d)}$$
$$K=0.1320, t_0=-1.5243 \qquad (d)'$$

In the two calculated steps above-mentioned, the two values of K have appeared. This is inevitable because the two kinds of models require the different data: the model (c) requires l_t but the model (d) requires (t, l_t), for $t=t_1—t_m$. In the first step the calculated $K_1=0.1315$ and $L_\infty=359$mm depend on each other, and in the second step $K_2=0.1320$ was aslo calculated with $L_\infty=359$mm as known. If $K_2=0.1320$ is chosen before $K_1=0.1315$, then the value of L_∞ is changed simultaneously, so that the $K_2=0.1320$ and $t_0=-1.5243$ lose the premise on which they can exist because they were evaluated with $L_\infty=359$mm as known in original second step. If $K_1=0.1315$ is chosen before $K_2=0.1320$, then the $t_0=-1.5243$ depending on the K_2 is changed at the same time. From this, what should be the value of t_0? This problem may be stated as that the t_0 is evaluated by using following revised Least-square method according to the model (d) with calculated L_∞ and K as knowns in "First,……L_∞ and K are evaluated according to……(c)."

In the $y=a+bx$ which is a revised model of linear regression with one independent variable, the slope b is known. The intercept a is selected to make Q attain minimum, here $Q \triangleq \sum_{i=1}^{n}(y_i-\hat{y}_i)^2$ which is the sum of squares of deviations between measured value y_i and its fitted value $\hat{y}_i=a+bx_i$ for $i=1-n$. Based on this principle, it has been derived that $a=\bar{y}-b\bar{x}$ which is Least-square estimated value of a. *From this, the previous formula* $(6)_2$, *which is used to compute* t_0, *was got.* Using the (♀) $K=0.1315$ and $L_\infty=359$mm in the article Yellowfin Sole, i. e. the former (c)′, $t_0=-1.5645$ has been computed according to the formula $(6)_2$ for $t=2$——15.

For the previous second calculating step which was recommended to the readers by the article Yellowfin Sole on page 65, its authors themselves do not know how to take instead. They had no alternative but to return toconform with the equality (4) in the reference "[2] Chen Dagang, etc., *Paralichthys Olivaceus*" on page 106 after the $K=0.1315$ and $L_\infty=359$mm have been evaluated according to the model "p65, (c)" in the previous first calculating step, and used the initial value condition $l_2=135$mm to calculate the value of t_0, so $t_0=-1.5869$ was found. As for "the t_0 is evaluated according to……(d)"……

Ⅳ. "K——Curvature Of The Growth Curves", Are The Growth Curves Both Circles (Or Circular Arcs) Whose Radii Equal 1/K?

Is the K in Von. Bertalanffy's growth functions the curvature of the growth curves expressed by them? *In the* advanced mathematics *textbooks for first year* at science-engineering universities, the curvature of a curve $y=f(x)$ is generally noted as K which is aslo a function of the independent variable x. This function takes only non-negative values and its definition field is the same as that of the function $y=f(x)$. The value of the function K at a point x stands for crooked level of the curve $y=f(x)$ at the point $(x, f(x))$. According to the mathematical formula about curvature, the curvature function K_{l_t} of von. Bertalanffy's body length growth curve $l_t=L_\infty[1-e^{-K(t-t_0)}]$ and the K_{w_t} of revised body weight growth curve $w_t=W_\infty[1-e^{-K(t-t_0)}]^b$ are respectively shown as follows:

$$\begin{cases} K_{l_t} = \dfrac{|l_t''|}{(1+l_t'^2)^{3/2}} = \dfrac{K^2L_\infty e^{-k(t-t_0)}}{[1+K^2L_\infty^2 e^{-2k(t-t_0)}]^{3/2}} & (7)_1 \\ K_{w_t} = \dfrac{|w_t''|}{(1+w_t'^2)^{3/2}} = \dfrac{bK^2W_\infty e^{-k(t-t_0)}[1-e^{-k(t-t_0)}]^{b-2}\cdot|be^{-k(t-t_0)}-1|}{\{1+b^2K^2W_\infty^2 e^{-2k(t-t_0)}[1-e^{-k(t-t_0)}]^{2b-2}\}^{3/2}} & (7)_2 \end{cases}$$

The K_{l_t}-curve and K_{w_t}-curve of yellowfin sole (♀), which include the assumed parts prolonged into juvenile stage, are shown in figure 5 and are explained auxiliarily by the following attached table. The figure 5 and the attached table were drawn on the basis of the values of the five growth parameters (K, L_∞, W_∞, b, t_0), which have been calculated in the article Yellowfin Sole.

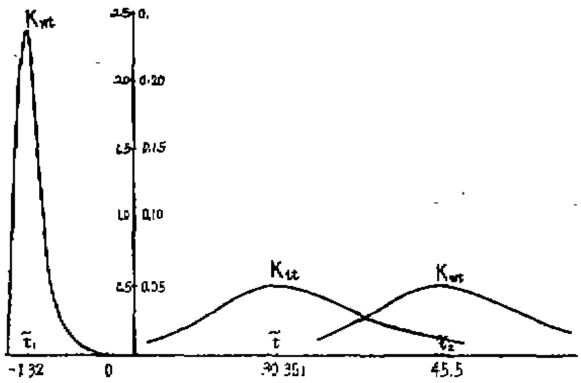

Fig. 5

t	……	t_0 −1.587	……	18	……	$\tilde{t}$ 30.061	……	48	……
K_{l_t}	0 ↑	0.000059	↑	0.009109	↑	0.050614373	↓	0.009076	↓ 0

(7)′₁

t	t_0 −1.587	−1.58	………	$\tilde{t}_1$ −1.32	………	0	………
K_{w_t}	0	0.096994	↑	2.380866145	↓	0.009094	↓

t	t' 6.709	………	34	………	$\tilde{t}_2$ 45.5	………	60
K_{w_t}	0	↑	0.011241	↑	0.050408969	↓	0.013589

(7)′₂

Notation: $\tilde{t}$——maximum point of the K_{l_t}-curve.

$\tilde{t}_1$——maximum point of the K_{w_t}-curve.

$\tilde{t}_2$——local maximum point of the K_{w_t}-curve.

t'——inflection point of the w_t-curve at which the curve turns from increasing 凹 pattern to increasing 凸 pattern.

It is easy to find the $K_{lt}<0.01$ and the $K_{wt}<0.01$ within the age interval $2\leqslant t\leqslant 15$ of the measured data. These calculated results indicate that the crooked levels of the l_t-curve and the w_t-curve at each point t are all very low and lower than that of a circle (or circular arc) whose radius $R=100$mm. The maximum values of the K_{lt}-curve and the K_{wt}-curve achieve separately at the point $\tilde{t}=30.061$ and the point $\tilde{t}_2=45.5$ which are possibly much larger than the age t_λ at which the fishes of one generation of yellowfin sole die as a result of senile decay. These two maximum values equal approximately 0.05, so are equivalent to the crooked level of a circle (or circular arc) whose radius $R=20$mm. The most crooked piece of the w_t-curve is the assumed part within the interval $[t_0, t_1)$ into which this curve is prolonged to juvenile stage.

It is known that curvatures at all points of a straight line equal identically zero, and these of a circle (or circular arc) equal identically reciprocal of this circle's radius. Except these two cases, curvature at a point of a curve is generally different from another of this curve, and curvature functions of different curves are different from one another. Hence the idea is a sheer fabrication that the curvature functions of the l_t-curve and the w_t-curve which are connecting to each other by the relation expression $w_t=a_w l_t^b$, both equal identically the same constant K.

Von. Bertalanffy's growth functions and the revision to w_t belong to physiological concept model. In the sense taking average for all individuals of one generation at every moment t and according to metabolism viewpoint, the increment of weight (biomass) of an organism is considered as the difference subtracting weight decrease as a result of catabolism from weight increase caused by anabolism. The first order differential equation $\frac{dl_t}{dt}=K(L_\infty-l_t)$ was deduced with the aid of the relation expression between l_t and w_t, $w_t=a_w l_t^3$ or $w_t=a_w l_t^b$; and the initial value condition $l_t|_{t=t_1}=l_1$ and the definition field $t_1\leqslant t\leqslant t_\lambda$ were suggested according to the physiological concept. Then the growth functions were derived by means of mathematical operations. The quotation being on page 65 of the reference "[2] Chen Dagang, etc., *Paralichthys Olivaceus*" reads "It is assumed that the physiological state of one generation is basically stable during the period from t_1 until t_λ, here t_1 is an age at which fishes of one generation have developed into overall sex-mature basically for the first time and t_λ is an age at which fishes die as a result of senile decay, and anabolism and catabolism of adult individuals are at a uniform speed. The 'uniform speed' means that assimilated quantity of nutritious substance on assimilating surface with a certain area of an individual and consumed (decomposed, excreted) quantity of nutritious substance by this individual with a certain body weight are both in direct proportion to time.

The uniform speeds, i. e. two coefficients of direct proportionality, are noted as the following two constants: H——coefficient of anabolism standing for the uniform speed of anabolism on assimilating surface with a unit area; and K——coefficient of catabolism standing for the uniform speed of catabolism in fish body with 1/b unit weight. In the sense taking average for all individuals of one generation at every moment t, the notations l_t, w_t and s_t are used to stand respectively for body length, body weight and assimilating surface area of one fish. Within a brief time interval $[t, t+dt]$ in which t standing for continuous age of adult fishes, increases from t to t + dt, the differential dw_t of increment of body weight w_t can be expressed as $dw_t = Hs_t dt - bKw_t dt$." This expression contains the following concepts in calculus: infinite dense cut, approximate replacement and taking linear principal part. And in the right end of this expression the s_t and w_t correspond to any moment t in the brief time interval $[t, t+dt]$. It is needed to list the following comparison:

$$dw_t = Hs_t dt - kw_t dt \quad * \quad dw_t = Hs_t dt - bKw_t dt$$

$$\left.\begin{matrix} w_t = a_w l_t^3 \\ s_t = a_s l_t^{3-1} \end{matrix}\right\} \rightarrow \quad * \quad \leftarrow \left\{\begin{matrix} w_t = a_w l_t^b \\ s_t = a_s l_t^{b-1} \end{matrix}\right.$$

$$3a_w l_t^{3-1} dl_t = l_t^{3-1}(Ha_s - ka_w l_t)dt \quad * \quad ba_w l_t^{b-1} dl_t = l_t^{b-1}(Ha_s - bKa_w l_t)dt$$

$$\frac{dl_t}{dt} = \frac{k}{3}\left(\frac{Ha_s}{ka_w} - l_t\right) \quad *$$

$$\text{let} \quad K = k/3 \quad *$$

$$\frac{dl_t}{dt} = K\left(\frac{Ha_s}{3Ka_w} - l_t\right) \quad * \quad \frac{dl_t}{dt} = K\left(\frac{Ha_s}{bKa_w} - l_t\right)$$

$$\lim_{l_t \uparrow L_\infty} \frac{dl_t}{dt} = 0$$

$$\frac{dl_t}{dt} = K(L_\infty - l_t)$$

$$l_t|_{t=t_1} = l_1 \qquad t_1 \leqslant t \leqslant t_\lambda$$

$$l_t = L_\infty - (L_\infty - l_1)e^{-K(t-t_1)} \qquad t_1 \leqslant t \leqslant t_\lambda$$

simplified form

$$w_t = W_\infty[1 - e^{-K(t-t_0)}]^3 \leftarrow l_t = L_\infty[1 - e^{-K(t-t_0)}] \rightarrow w_t = W_\infty[1 - e^{-K(t-t_0)}]^b$$

$$t_1 \leqslant t \leqslant t_\lambda$$

As mentioned above, the inductive process based on the physiological concept and the mathematical deriving process, in which the former is foundation of the latter, for Von. Bertalanffy's growth functions and the revision to w_t, indicate very clearly that K is coefficient of catabolism standing for the uniform speed of catabolism in fish body with 1/3 unit weight, or more generally, with 1/b unit weight. The magnitude of K-value is a sign of exuberant level of metabolism of an organism.

The K in Von. Bertalanffy's growth functions is called generally as growth coefficient in the research field of international fish ecology at the present time. This name came perhaps from the former differential equation $\frac{dl_t}{dt}=\frac{Ha_s}{3a_w}-Kl_t$, in which the term $(-K)l_t$ is added to the constant $\frac{Ha_s}{3a_w}$ to get $\frac{dl_t}{dt}$ ——instantaneous growth speed at moment t of average body length l_t. It is necessary to indicate that the slope $(-K)$ in linear relation expression $\frac{dl_t}{dt}=\frac{Ha_s}{3a_w}-Kl_t$ can be called as coefficient of l_t which is growing. Please take notice to that the former slope, i. e. the so-called "growth coefficient", is not the K, so a question is just revealed here. About the word "coefficient", please consult THE ADVANCED LEARNER'S DICTIONARY ENGLISH-ENGLISH -CHINESE OXFORD UNIVERSITY PRESS Tenth impression 1978 on page 196, which reads Co-efficient n. 1. (maths.) number or symbol placed before and multiplying another quantity, known or unknown. (In 3xy, 3 is the ~of xy.)

It is necessary to use some letters more or less as notations of numbers and quantities or other objects to be studied in every research field of natural science. Especially in the research fields involving mathematical contents, English letters are more indispensable. It is a very common and elementary knowledge that a same letter stands for different connotations and has corresponding terms to those in different mathematical problems. Take an example for English letter K:

1) In research of differential calculus about curves, K stands for crooked level of a curve at every point within its definition field and so is called as curvature of a curve.

2) In Logistic reproduction model about populations, K stands for increasing limit of a population's number and quantity (or their densities), which the environment is able to accommodate, and so is called as environmental carrying capacity.

3) In reproduction model $N_t=N_0 2^{kt}$ about binary fissiparous cell, K stands for the number of times for cells dividing within per unit time and so is called as fission frequency of cell.

4) In Von. Bertalanffy's growth functions and the revision to w_t about generations, K stands for uniform speed of catabolism in fish body with 1/3 unit weight, or more generally, with 1/b unit weight, and so is called as coefficient of catabolism in the derived process of Bertalanffy's growth functions and the revision to w_t. And so forth and so on.

How can a name in one research field be indiscriminately imposed on another only because these names have the same letter used as their notations?

In biomathematical research field we should forbid turning mathematical terms upside-down.

Reference

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〔2〕宋云升，陈大刚，Jan.,1989，东白令海的刺黄盖鲽渔场海洋学与渔业生物学特征。海洋科学(MARINE SCIENCES)，1989，No.1，45-48p.

〔3〕宋云升，陈大刚，Sep.，1989，东白令海刺黄盖鲽（*Limanda aspera*）渔业。远洋渔业(OVER SEA FISHERIES)，1989，No.3，9-14p.

〔4〕宋云升，陈大刚，Sep.,1989，东白令海刺黄盖鲽(*Limanda aspera*)年龄与生长的调查研究。海洋湖沼通报，1989，No.3，63-70p.

摘　要

生长参数的生物学概念

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本文由互不依赖的四篇短文组成，相应地，本文的摘要划分为四点主要结论。

本文提出生长参数的生物学概念问题，与《东白令海刺黄盖鲽(*Limanda aspera*)年龄与生长的调查研究》[4]一文的作者商榷，其中包括所谓的"生长指标"以及Von.Bertalanffy生长函数及其修改中的生长参数b，t_0与K。

四点主要结论如下：

1．生长指标(growth indexes)$D_l \triangleq l_t \ln\frac{l_{t+1}}{l_t}$和$D_w \triangleq W_t \ln\frac{W_{t+1}}{W_t}$与生长快慢本身具

的生态概念背道而驰，是对于相对生长率(relative growth rates) $C_{lt} \triangleq \frac{l_{t+1}-l_t}{l_t}$和 $C_{wt} \triangleq \frac{w_{t+1}-w_t}{w_t}$的否定，不能作为划分生长阶段的依据。《刺黄盖鲽》一文中的一系列自相矛盾的计算结果清清楚楚地暴露了所谓的生长指标实际上指不了什么标，它是在数学外衣掩盖下的谬误，应该向世界各国的鱼类生态学界建议废除。

2. 鱼类种群一个世代在时刻 t 的平均体长l_t和平均体重w_t之间的幂函数关系模式 $w_t = a_w l_t^b$中的生长参数 b 是异速生长常数(constant of allometry growth)，表示w_t按其对数值的生长量(以及速度)是对应的l_t按其对数值的生长量(以及速度)的 b 倍。归纳概括出上述的关系式依赖于下列假设："鱼体均匀生长，即体形与比重(密度)不变。"这一假设不涉及b值的大小，与b值是否等于3无关，根本不是"如果b＝3"的推论，而是人们对于实际情况的一种近似。

3. Von. Bertalanffy 生长函数及其修改中的 t_0 是一个假定的年龄(an assumed age)，是假定生长曲线所描述的成年鱼的生长规律也适用于幼鱼而将其向幼鱼阶段延伸与 t 轴的交点的横坐标。这样延伸出一个假定的对应于平均体长和平均体重等于零的年龄 t_0 仅仅是为了简化生长函数作为一种数学模式的表达形式，根本不涉及任何生物学理论和数学理论，也根本不存在不知什么样的理论研究得出的所谓体长和体重等于零时的理论年龄(theoretical age at which length and weight of fish body equal zero)。

4. 关于 Von. Bertalanffy 生长函数及其修改的生理学概念归纳过程以及在此基础上的数学推导过程，清清楚楚地指明了生长函数中的生长参数K是异化作用系数(coefficient of catabolism)，它表示$\frac{1}{3}$单位重量，或者一般地，$\frac{1}{b}$单位重量的鱼体中异化作用的匀速度；而在关于曲线的微分学研究中字母K用作表示曲线弯曲程度的曲率的记号。不能由于用作记号的字母相同就把异化作用系数K称为生长曲线的曲率(curvature of the growth curves)，从而导致体长和体重两条生长曲线都成了半径等于$\frac{1}{K}$的圆周(或者圆弧)。在生物数学研究领域应该禁止乱套数学术语。